Optimal reconfiguration of the power distribution system

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Abstract - The main objective of the optimal reconfiguration of electrical distribution system is to minimize the active power losses of the network. Finding the optimal topology of the grid by taking into account physical and operational constraints can be formulated as an optimization problem. Topology of the distribution grid must also be taken into account, as the system is usually built as meshed, but operated as radial system which represents the constraint to the problem formulation.

Distribution System Reconfiguration (DSR) utilizes the power flow equations to model the distribution grid while taking into account current state of the system. In order to codify the reconfiguration functionality of the reconfiguration the model uses with binary variables of the switching states. Furthermore to better model the behavior of the distribution system the DSR formulations are extended with dynamic models of system elements. The report summarizes the recent work in the field of optimal reconfiguration of power distribution system.

Keywords— optimal power flow, power distribution system, distribution system reconfiguration, radiality constraint, linear programming, convex relaxation

I. INTRODUCTION

Minimization of active power losses in electrical distribution system has been recently in the focus of research of the power community. Latest research shows that around 8% are the losses on the power lines of the transmission and distribution system [1]. The data about transmission and distribution losses is given in Figure 1 for the period of 2000 - 2011. The basic idea is to find the distribution network configuration (topology) that will ensure minimal (active) power losses while taking into account physical, environmental and operational constraints. In general, this problem - the optimal Distribution System Reconfiguration (DSR) - can be formulated as an optimization problem.

To solve the optimal DSR problem (that results in minimal power losses) one needs a mathematical model of the distribution system. An efficient and exact modelling of power system is based on power flow equations. The optimal operating point of the power system is found by solving the optimal power flow (OPF) problem, which is the fundamental problem in electrical energy systems analysis [2]. Due to the system constraints, the OPF problem is a nonconvex problem that is rather hard to solve. The latest convex relaxation methods enabled formulation of the OPF problem as a convex second order-cone program (SOCP) that can be efficiently solved by readily available solvers. With additional approximation one can convert the SOCP formulation to a linear program (LP) that can be solved even more efficiently and reliably [3].

Fig. 1. Electric power transmission and distribution losses

To utilize the power flow equations that resulted from modeling of the distribution system, they have to be extended in order to satisfy the basic DSR concept. Firstly, to codify reconfiguration functionality of distribution network, the system model must be extended with binary variables that describe the switching status of the lines of the network. Secondly, model extension must take into account the basic operational constraint of the distribution networks. Namely, even though distribution networks are physically built as meshed and interconnected, they are operated as radial networks, since such configuration allows for simplified design and more reliable operation of protection systems. One should point out that the radiality constraint is not easy to incorporate into the OPF problem formulation.

Radiality problem is similar to the minimum spanning tree (MST) problem which has been analyzed extensively in literature [4], [5], [6]. A direct analogy between radiality constraint of the DSR problem and the MST problem is readily established in the case of planar networks. Unfortunately one cannot guarantee that the general distribution systems are planar, especially in modern time when there is a lot of distributed generation. The incorporation of the radiality constraint must be available also for the general network.
topology so this has to be taken into account in the case of the OPF problem extension.

Once the extension of the OPF problem has been reached the problem formulation can be used for loss minimization analysis. Some authors [7] have shown in simulations that around 10% loss reduction can be achieved by the use of DSR.

In order to achieve a more realistic DSR the distribution system must be modeled to include dynamics of all the elements of the system (generators, energy storage and distributed active loads).

The rest of this report is organized as follows: a brief overview of the distribution networks is given in Section II. Section III describes the OPF formulation used for further analysis. Formulation of the DSR problem based on OPF extension with radiality constraints and introduction of reconfiguration functionality is given in Section IV of the report. Application of the static OPF and DSR problems and their extension towards inclusion of dynamics are elaborated in Section V. Final remarks and directions for future work are presented in Section VI.

II. DISTRIBUTION SYSTEMS - FUNDAMENTALS

The electrical power system is one of the largest coupled systems developed by mankind. It can be split into three subsystems: generation system, transmission system and distribution system. Electrical power produced by the generation system is being delivered to the end users by electrical networks: transmission and distribution networks. While transmission system is in charge of transporting large power blocks over the countries and regions, distribution system is in charge of delivering and distributing this power to the end customers. Transmission systems utilize High Voltage (HV) grids for power transmission, while distribution systems utilize Medium Voltage (MV) and Low Voltage (LV) grids for the task of delivery of electricity to every small part of the network.

Distribution network usually includes medium voltage power lines (10 to 35 kV), substations (110/x, 35/x, 20/x, 10/x) and low voltage lines (less than 1 kV). Having in mind total number of power lines, it is clear that the distribution system is larger than the transmission one, and that it usually has a more complex structure. Two types of network physical realization characterize the distribution system: radial networks and interconnected networks. Radial networks are based on radial feeders from primary substations through network area without any reserve connection from any other point of supply. Radial type networks are typical for remote and isolated rural areas on the end of the feeders. On the other hand interconnected networks have mashed feeders that allow power supply from more available points that are interconnected and are characteristic for more urban areas.

Even though distribution operators tend to build more interconnected networks, because of the benefits of multiple supply routes, these networks are almost always operated radially. The points of connection are normally open points and could be closed to form a ring structure thus allowing alternative supply route. Radial operation of distribution networks allows easier operation in case of faults in the network or maintenance procedures. In such cases a small part of the network could be isolated while other parts would be available for power restoration.

Operation of the distribution system is today done remotely from the control center. Advances in ICT technology allow comfortable operation of a large distribution system through Distribution Management Systems (DMS) installed in control centers of Distribution System Operator (DSO). DMS utilize SCADA and telecommunication systems and infrastructure for remote control of substations on all voltage levels of the network. On the other end of the control chain are local Intelligent Electrical Devices (IED) that are installed on substations with functions of protection, measurement and control.

An ever-increasing demand for reliable and high quality electric power brought to attention the so-called smart grid concepts [8]. Instead of unidirectional power flow of the classical electricity system from large generation units to the end-customer, in smart grid bidirectional power flow allows for distributed generation that is connected on the LV side of the system. Interface for these small distributed generation units are MV and LV networks and can inject power that could be sufficient for local consumption or even available for power export to the network.

Fig. 2. Smartgrid concept of distribution grid

Smart grid concept are a prerequisite to achieving the EU’s ambitious energy and climate objectives to 2020 and beyond – decreasing greenhouse gas emissions, increasing energy efficiency and the strongest driver of all, increasing the share of renewable energy connected to power network [9].

Taking into account all the above mentioned aspects, a rather complex task is set on the DSO to enable reliable and safe power supply to the end customer, ensuring delivery of high quality power and reducing operational cost through system management and maintenance.

III. OPTIMAL POWER FLOW

The power system analysis is synonymous with the (numerical) analysis of the power flow in electrically interconnected systems. Power flow focuses on the aspects of AC power parameters of the systems thus analyzing the normal steady state operation. Power flow analysis is today commonly used for power system planning, control and operations thus providing more information regarding the state of the system. Analysis gives an insight on the power system states that are affected by operational conditions and contingency situations.

The optimal power flow problem refers to the problem of finding the optimal electrical power network operating point
that ensures security, reliability and affordability while satisfying a set of system constraints. These constraints are usually limitations of physical, environmental and operating nature to the system. First problem formulations date back to 1962 and have since been recognized as a fundamental problem regarding power system analysis [10]. The OPF problem is widely used and is a base for many applications such as economic dispatch, unit commitment, state estimation, stability and reliability assessment, volt/var control and demand response [2].

OPF problem is a mathematical program that seeks to minimize a certain function such as total power loss of the system, total generation cost of the generation units or customer outage time subject to Kirchoff’s laws and constraints.

The OPF problem is very difficult to solve for AC power networks because of two reasons:

1. nonlinear inequality due to nonlinear Kirchoff’s equations that govern the power flows in electrical networks,
2. nonconvex constraints mainly due to magnitude of complex valued bus voltages.

In order to tackle these problems there are basically three common solutions:

1. use linear approximations of power flow equations (DC OPF) [2],
2. employ the nonlinear solvers to find a local optimum of the OPF problem [11],
3. exploit convex relaxation of nonconvex constraints [12].

Linearization of exact power flow equations is only justified once the power losses are small and the bus voltages are close to nominal values while the voltage angle difference between buses are small. This is the case for transmission networks since in such networks the resistance of the lines is much smaller than the reactance and the voltage fluctuations are inconceivable [13]. Completely different situation is for distribution networks where the losses are much higher and voltage fluctuates considerably. Limitations of DC OPF are that the solution of the problem may diverge from the real OPF solution and it may even not be feasible. Since this report is considering optimal network reconfiguration of the distribution system, this method won’t be considered any further.

To tackle the limitations of DC OPF, a nonlinear algorithm may be employed to seek a local optimum. However, such approach does not guarantee convergence in all cases. Even if the solution converges to a local optimum, no information about the global optimum is available, thus making this method unsatisfactory.

Convex relaxation approach offers several advantages compared to other mentioned methods. The convexified OPF can today be solved rather easily using various freely available solvers (MOSEK [14], SeDuMi [15] or SDP3 [16]). In case of the exact relaxation a certified global solution is recovered due to convexity of the problem. In case of inexact relaxation, the recovered solution provides a lower bound on the minimum cost which can then be used to verify the distance of the feasible solution to the global optimality. Also if the relaxation is infeasible, the original OPF problem is also infeasible [17].

A. OPF problem formulation

Consider a power network represented by the graph $G = (V, E)$ and the set of generator buses $G \subseteq V$, where $V = \{1,2,...,n\}$ is the set nodes and $E \subseteq V \times V$ is the set of flow lines $(i,j)$, where $i, j \in V$ and $i \neq j$.

Let $N(i)$ denote a set of all nodes adjacent to node $i$. $N(i) = \{j | i, j \in E\}$. Node $n$ is designated as the root of the network and represents the substation node. The substation node connects the distribution network to the rest of the power system. This node is used to balance the active and reactive power in the network and is known in literature as “slack bus”, hence $n \in G$.

We define the following variables and parameters of the system model:

- $P_i^D$ and $Q_i^D$ as active and reactive power of the load connected to node $i \in V$ ($P_i^D = Q_i^D = 0$ when there is no load at node $i$),
- $P_i^G$ and $Q_i^G$ as active and reactive power of the generator connected to node $i \in V$ ($P_i^G = Q_i^G = 0$ when $i \in V \setminus G$),
- $V_i$ the voltage magnitude at node $i \in V$,
- $\theta_i$ the voltage angle at node $i \in V$,
- $\theta_{ij}$ the voltage angle difference between nodes $i$ and $j$, $(i,j) \in E, \theta_{ij} = \theta_i - \theta_j$,
- $P_{ij}$ and $Q_{ij}$ as active and reactive power transferred from node $i \in V$ to the rest of the network through line $(i,j) \in E$.

According to [18] the circuit model of the power network can be derived by replacement of every power line and transformer by equivalent Π model. In the circuit model, for each line $(i,j) \in E$, one can define the following:

- $z_{ij}$, the line impedance with $r_{ij} = \text{Re}\{z_{ij}\}$ and $x_{ij} = \text{Im}\{z_{ij}\}$,
- $y_{ij} = z_{ij}^{-1}$, the line admittance with $g_{ij} = \text{Re}\{y_{ij}\}$ and $b_{ij} = \text{Im}\{y_{ij}\}$.

For every node of the distribution network $i \in V$, the following constraints on active and reactive power injection ($P_i^D$ and $Q_i^D$ respectively) must be ensured:

$$P_i^D = P_i^G - P_i^L = \sum_{j \in N(i)} [g_{ij}v_i^2 - \sqrt{3}v_i(g_{ij} \cos \theta_{ij} + b_{ij} \sin \theta_{ij})], \forall i \in V. \quad (1)$$

$$Q_i^D = Q_i^G - Q_i^L = \sum_{j \in N(i)} [-b_{ij}v_i^2 - \sqrt{3}v_i(g_{ij} \cos \theta_{ij} - b_{ij} \sin \theta_{ij})], \forall i \in V. \quad (2)$$

Due to the generation constraints the active and reactive power generation is bounded as follows:

$$P_i \leq P_i^G \leq P_i^L, \quad \forall i \in V, \quad (3)$$
\[ Q_i \leq Q_i^l \leq \bar{Q}_i, \quad \forall i \in \mathcal{V}. \quad (4) \]

Network operational constraints, that limit the deviation of voltage magnitude, constrain it respectively:
\[ V_i \leq V_i \leq \bar{V}, \quad \forall i \in \mathcal{V}. \quad (5) \]

The network active power losses are equal to the difference between the total network active power generation and the total network active power consumption. Consequently the network active power losses can be derived as a sum of all injections (6) at all nodes of the network:
\[ P_{\text{loss}} = \sum_{i \in \mathcal{V}} P_i^l \quad (6) \]

The OPF problem, that minimizes active power losses, can now be formulated as:
\[
\begin{align*}
\min_{v, P, Q, \alpha} & \quad \sum_{i \in \mathcal{V}} P_i^l \\
\text{s.t.} & \quad P_i^1 = P_i^G - P_i^D = \sum_{j \in \mathcal{N}(i)} \left[ g_{ij} V_i^2 - V_i V_j (g_{ij} \cos \theta_{ij} + b_{ij} \sin \theta_{ij}) \right], \forall i \in \mathcal{V}, \\
& \quad Q_i^1 = Q_i^G - Q_i^D = \sum_{j \in \mathcal{N}(i)} \left[ -b_{ij} V_i^2 - V_i V_j (b_{ij} \cos \theta_{ij} - g_{ij} \sin \theta_{ij}) \right], \forall i \in \mathcal{V}, \\
& \quad P_i \leq P_i^G \leq \bar{P}_i, \quad \forall i \in \mathcal{V}, \quad (10) \\
& \quad Q_i \leq Q_i^G \leq \bar{Q}_i, \quad \forall i \in \mathcal{V}, \quad (11) \\
& \quad V_i \leq V_i \leq \bar{V}, \quad \forall i \in \mathcal{V}. \quad (12)
\end{align*}
\]

Given formulation of the OPF problem aims to find the optimal active and reactive power references for all the generators in the network which will ensure the supply of energy to all consumers with minimum active power losses and all voltages kept safely in predefined interval and satisfying operating and physical constraints.

The optimization problem as given by (7) is nonconvex and thus rather hard to solve. Thanks to the recent developments in the field of convex optimization, the OPF problem formulated in (7) and defined as a nonconvex, can easily be relaxed to a convex problem as a second-order cone program.

### B. Convex relaxation of the OPF problem

Convex optimization [17] is a special class of mathematical optimization problems that includes, as special cases, least-squares and linear programming problems. These two methods arise in a variety of applications and theory of solving such problems is well known and numerical methods are very efficient in finding solutions of such problems.

Recent developments in convex optimization field, such as introduction of interior-points methods, has made possible to solve semidefinite programs and second-order cone programs in polynomial time, i.e. very efficiently.

According to [3] convex relaxation of the OPF problem formulation can be relaxed by introduction of the following new variables:
\[ v_i = \frac{V_i^2}{\sqrt{2}}, \quad \forall i \in \mathcal{V}, \quad (13) \]
\[ R_{ij} = V_i V_j \cos \theta_{ij}, \quad \forall (i, j) \in \mathcal{E}, \quad (14) \]
\[ T_{ij} = V_i V_j \sin \theta_{ij}, \quad \forall (i, j) \in \mathcal{E} \]

one can formulate convex relaxation of the OPF problem as the following SOCP problem:
\[
\begin{align*}
\min_{v, R, T, P, Q, \alpha} & \quad \sum_{i \in \mathcal{V}} P_i^l \\
\text{s.t.} & \quad P_i^1 = P_i^G - P_i^D = \sum_{j \in \mathcal{N}(i)} \left( \sqrt{2} g_{ij} v_i - g_{ij} R_{ij} \right), \forall i \in \mathcal{V}, \\
& \quad Q_i^1 = Q_i^G - Q_i^D = \sum_{j \in \mathcal{N}(i)} \left( -\sqrt{2} b_{ij} v_i + b_{ij} T_{ij} \right), \forall i \in \mathcal{V}, \\
& \quad 2v_i v_j \geq R_{ij}^2 + T_{ij}^2, \quad \forall (i, j) \in \mathcal{E}, \\
& \quad R_{ij} \geq 0, \quad \forall (i, j) \in \mathcal{E}, \quad (20) \\
& \quad P_i \leq P_i^G \leq \bar{P}_i, \quad \forall i \in \mathcal{V}, \quad (21) \\
& \quad Q_i \leq Q_i^G \leq \bar{Q}_i, \quad \forall i \in \mathcal{V}, \quad (22) \\
& \quad \frac{v_i^2}{\sqrt{2}} \leq v_i \leq \frac{\bar{V}_i^2}{\sqrt{2}}, \quad \forall i \in \mathcal{V}. \quad (23)
\end{align*}
\]

To guarantee that the solution to the SOCP problem is globally optimal, the convex relaxation must be exact. Exactness of global solutions is analyzed in detail in [20], and for the convex SOCP relaxation of the OPF problem will be met only if the cone constraint (19) is active at the optimal point. This condition can easily be check once the optimization process has finished.

Convex relaxation of the OPF problem, defined as SOCP can easily be solved by some of the available solvers. Performance and reliability of such solvers are still questionable today. One of the ways to tackle this problem is to use polyhedral approximation of the second-order cone, resulting in a linear program formulation. LP problem
IV. DISTRIBUTION SYSTEM RECONFIGURATION - DSR

Radial operation of the network is done by opening and closing line switches of the feeders from central SCADA system. Meshed interconnected distribution network is consisted of normally closed switches called sectionalizing switches and normally open switches called tie switches. Tie switches are normally open in order to maintain radial structure of the network. In case of a fault on the feeder, sectionalizing switches are used to isolate the fault part of the network, and restore supply by closing the tie switch. In any case the radial operation of the distribution network must be maintained.

Studies have shown that at peak operation 5% of generated power is lost in line losses of distribution system [2]. Today in the era when energy efficiency is in the main focus, researchers are trying to find a way of efficient and least expensive way of power generation, transmission and distribution. One of the most effective methods of loss minimization in distribution networks is network reconfiguration. Distribution system reconfiguration allows transfer of loads by operation sectionalizing and tie switches from heavily loaded feeders to lightly loaded feeders. Result of such operation is minimization of active power loss of the distribution system.

DSR must also take into account constraints of the system such as operational limits and radially. Radial structure of distribution network must be contained in order for the protection devices to remain properly coordinated.

Beside power loss minimization, DSR can also have other objectives such as service restoration, load balancing and voltage profile improvement [21].

In the case of power loss minimization, one can define that the objective of DSR is to find radial topology of the distribution network that connects all nodes in the network and ensures minimal active power losses over the distribution lines while satisfying operating conditions. This objective will be the subject of analysis in this paper and in future work.

As many authors have argued it is quite clear that the DSR problem can be formulated as a mixed-binary nonlinear optimization problem [7]. Binary variables represent the switch statuses and the continuous variables model the power distribution network.

Intuitive solution of before formulated DSR problem would be consisting of couple of steps:

1. enumerate all the possible network configurations,
2. solve the OPF problem for each of the configurations,
3. choose the one with the minimum active power losses as the problem solution.

Disadvantage of such intuitive, brute-force approach is highly impractical because of the number of possible network configurations for that one needs to solve the OPF problem. The number of possible configurations, even for moderate size networks, grows exponentially with the size of the network.

Some of the first methods of solving the OPF problem in order to minimize losses have been based on heuristic approach. Based on the before mentioned naive approach to problem solving, one method starts with all the switches in the network closed, thus having the complete network meshed. In each iteration of the method, switches are being opened sequentially in order to minimize the network active power losses. The solution is obtained once a radial network structure is obtained thus giving a minimal active power topology solution of the OPF problem [21].

Other early heuristic, but still an advanced method starts from the radial configuration of the network. In each iteration an operation consisting of closing of a tie switch and opening of a sectionalizing switch is done in order to maintain radial structure. This operation is called branch exchange and continues until optimal solution of the topology is found [22]. Although these early heuristic methods are fast, none of them guarantees global optimality of the solution. This was the biggest disadvantage of early heuristic methods, thus resulting in continued research for better methods of solving the OPF problem.

Other modern approaches try to employ formal optimization methods such as stochastic and deterministic methods. Stochastic methods are rather easy to implement but their results are generally not repeatable. In some cases these stochastic methods may require several runs of the algorithm in order to obtain a satisfactory solution which can result in great computational cost. The global optimality is still not guaranteed and cannot be formally verified which makes these methods also undesirable. Examples of such methods are based on metaheuristics and artificial intelligence techniques and include simulated annealing technique [23] and genetic algorithms [24].

Deterministic methods have been in focus recently but have not still been fully exploited. Some authors propose linearization of the DSR optimization problem and present the problem as a mixed-integer linear program [4]. Some authors model loads as constant current sources with constant impedance, and formulate the DSR problem as mixed-integer quadratic program. If convex relaxation is applied network reconfiguration problem can be formulated as a mixed-integer second order cone program as in [7].

A. Extension of the OPF problem formulation with reconfiguration ability

In order to be able to solve the DSR problem efficiently, one must extend the given SOCP relaxation of the OPF problem by imposing the reconfiguration ability and radiality constraint as discussed before.

To ensure reconfiguration of the network the OPF problem has to be extended by introduction of new binary variables that describe the status of the switches. New binary variable $\delta_l$ is defines for each line $v(i,j) \in E$, where $\delta_l$ indicates switching status of the line, thus meaning $\delta_l = 1$ the line is closed and $\delta_l = 0$ line is opened.

Moreover we define the variables $v^l_i$ and $v^l_j$ for each line $l$ associated with its endpoints $i$ and $j$ respectively. These
variables need to be set to zero when the line is disconnected ($\delta_i = 0$) and take the values at the nodes $i$ and $j$ ($v_i$ and $v_j$ respectively) when the line is connected ($\delta_i = 1$). To ensure this following inequalities are introduced:

$$0 \leq v_i^j \leq \frac{\bar{v}_p^2}{2\delta_p}, \quad \forall l \in \{1, 2, ..., m\},$$

$$0 \leq v_i^j \leq \frac{\bar{v}_p^2}{2\delta_p}, \quad \forall l \in \{1, 2, ..., m\},$$

$$0 \leq v_i - v_j \leq \frac{\bar{v}_p^2}{2}(1 - \delta_l), \quad \forall l \in \{1, 2, ..., m\},$$

$$0 \leq v_j - v_i \leq \frac{\bar{v}_p^2}{2}(1 - \delta_l), \quad \forall l \in \{1, 2, ..., m\},$$

By introduction of the new binary variables that describe the switching status of the line we can formulate the following mixed-integer second order cone program (MISOPC) with the ability to reconfigure the topology of the network:

$$\min_{v, r, t, pG, qG, k} \sum_{i \in V} p_{i}^l$$

s.t.

$$p_i^l = p_0^l - p_i^l = \sum_{j \in n(i)} p_{ij} = \sum_{j \in n(i)} (\sqrt{2}b_{ij}v_i^j - g_{ij}R_{ij} + h_{ij}t_{ij}), \forall i \in V,$$

$$q_i^l = q_0^l - q_i^l = \sum_{j \in n(i)} q_{ij} = \sum_{j \in n(i)} (-\sqrt{2}b_{ij}v_i^j + g_{ij}R_{ij} - h_{ij}t_{ij}), \forall i \in V,$$

$$2v_i^j v_j^i \geq R_{ij}^2 + T_{ij}^2, \forall (i, j) \in E,$$

$$R_{ij} \geq 0, \forall (i, j) \in E,$$

$$P_i \leq P_{i}^0 \leq \bar{P}_i, \forall i \in V,$$

$$Q_i \leq Q_{i}^0 \leq \bar{Q}_i, \forall i \in V,$$

$$0 \leq v_i^j \leq \frac{\bar{v}_p^2}{2\delta_p}, \forall l \in \{1, 2, ..., m\},$$

$$0 \leq v_i^j \leq \frac{\bar{v}_p^2}{2\delta_p}, \forall l \in \{1, 2, ..., m\},$$

$$0 \leq v_i - v_j \leq \frac{\bar{v}_p^2}{2}(1 - \delta_l), \forall l \in \{1, 2, ..., m\},$$

$$0 \leq v_j - v_i \leq \frac{\bar{v}_p^2}{2}(1 - \delta_l), \forall l \in \{1, 2, ..., m\},$$

$$\frac{\bar{v}_p^2}{\sqrt{2}} \leq v_i \leq \frac{\bar{v}_p^2}{\sqrt{2}}, \forall i \in V,$$

$$\delta_i \in \{0, 1\}, \forall i \in \{1, 2, ..., m\}.$$}

**B. Extension of the OPF problem formulation with radiality constraint**

Radial topology usually refers to the configuration that includes all the nodes but doesn’t have any loops. In heuristic methods, radiality is usually dealt with implicitly, while in direct mathematical models a mathematical formulation for the radiality constraint is required.

Authors in [25] define the two conditions that have to be met in order to establish a radial topology:

1. All the nodes are inside a subgraph,
2. The subgraph is connected and has no loops (simple cycles).

The basic assumption is that the distribution network may be modeled as an undirected graph, taking its nodes as vertices and its branches as edges. The first condition ensures that the subgraph spans all the nodes and the second condition ensures the subgraph is a tree. These two are necessary and sufficient conditions.

Some of the authors argue that the sufficient radiality constraint is that the topology has $n-1$ branches, where $n$ is the number of nodes. Counterexamples in [5] show that this condition is not sufficient for ensuring radiality.

Another property of a tree is that for every node in the tree there exists a unique path from that node to the root node. Hence, a naive approach of ensuring radiality would be to enumerate all possible paths from every node to the root node and require that only one path is selected. Clearly this approach is highly impractical because the number of such possible paths grows exponentially with the size of the network. This exponential growth of the number of radiality constraints is inherited from the MST problem.

**V. DYNAMIC DISTRIBUTION SYSTEM RECONFIGURATION - DDSR**

In order to better evaluate the behavior of a distribution network both OPF and DSR may be extended in such way that the dynamics of system elements are integrated into static OPF and DSR formulations.

Dynamics are assumed for generators (denoted with G), energy storage (denoted with S) and loads (denoted as D). For each of devices the following linear dynamics can be assumed:

$$x_G^i(t) = A_G x_G^i(t) + B_G u_G^i(t),$$

$$x_S^i(t) = A_S x_S^i(t) + B_S u_S^i(t),$$

$$x_D^i(t) = A_D x_D^i(t) + B_D u_D^i(t),$$

where $x_i \in \mathbb{R}^{n_i}$ denotes the state vector with $n_i$ states, $u_i \in \mathbb{R}^{n_u}$ denotes the input vector with $n_u$ inputs. At each node of the distribution system and combination of generators, energy storage and loads can be connected.

State vectors and input vectors are a subset of feasible sets which represent the constraints that can be defined as:

$$x_G^i \in X_G^i \subseteq \mathbb{R}^{n_G}, \quad u_G^i \in U_G^i \subseteq \mathbb{R}^{n_u},$$

Hence, a naive approach of ensuring radiality would be to ensure that certain conditions are satisfied. These two are necessary and sufficient conditions.

which represent the constraints that can be defined as:
\[ x^S_i \in \mathbb{R}^n, \quad u_i^S \in \mathbb{R}^m, \quad x^D_i \in \mathbb{R}^n, \quad u_i^D \in \mathbb{R}^m. \]  
\[ x^S_i \subseteq \mathbb{R}^n, \quad u_i^S \subseteq \mathbb{R}^m, \quad x^D_i \subseteq \mathbb{R}^n, \quad u_i^D \subseteq \mathbb{R}^m. \]

If a certain device is not connected to the node, then its feasible sets are empty sets and the dynamics matrices are zero as well.

Active and reactive power for each of distribution system elements can be modeled and define as an affine function of its states and inputs, e.g. for a generator:

\[ P_i^G(t) = C_i^G x_i^G(t) + D_i^G u_i(t) + P_{0,i}^G(t), \]
\[ Q_i^G(t) = E_i^G x_i^G(t) + F_i^G u_i(t) + Q_{0,i}^G(t). \]

After defining the system dynamics we can define the aggregated dynamics for each node of the distribution system:

\[ x_i(t+1) = A_i x_i(t) + B_i u_i(t), \]

where

\[ x_i(t) = \begin{bmatrix} x_i^G(t) \\ x_i^D(t) \end{bmatrix}, \quad u_i(t) = \begin{bmatrix} u_i^G(t) \\ u_i^D(t) \end{bmatrix}, \]
\[ A_i = \text{diag}(A_i^G, A_i^D), \quad B_i = \text{diag}(B_i^G, B_i^D). \]

Similarly the aggregated active and reactive power injection vectors can be defined as follows:

\[ P_i(t) = C_i x_i(t) + D_i u_i(t) + P_{0,i}(t), \]
\[ Q_i(t) = E_i x_i(t) + F_i u_i(t) + Q_{0,i}(t). \]

where

\[ P_i(t) = \begin{bmatrix} P_{i}^G(t) \\ P_{i}^S(t) \end{bmatrix}, \quad Q_i(t) = \begin{bmatrix} Q_{i}^G(t) \\ Q_{i}^S(t) \end{bmatrix}, \]
\[ C_i = \text{diag}(C_i^G, C_i^D), \]
\[ D_i = \text{diag}(D_i^G, D_i^D), \]
\[ P_{0,i}(t) = \begin{bmatrix} P_{0,i}^G(t) \\ P_{0,i}^S(t) \end{bmatrix}, \quad P_{0,i}^D(t), \]
\[ E_i = \text{diag}(E_i^G, E_i^D), \]
\[ F_i = \text{diag}(F_i^G, F_i^S, F_i^D), \]
\[ P_{0,i}(t) = \begin{bmatrix} P_{0,i}^G(t) \\ P_{0,i}^S(t) \end{bmatrix}, \quad P_{0,i}^D(t), \]
\[ Q_{0,i}(t). \]

The aggregated state vectors and input vectors are then defined as:

\[ x_i \in \mathcal{X}_i = \mathcal{X}_i^G \times \mathcal{X}_i^D \times \mathcal{X}_i^P, \]
\[ u_i \in \mathcal{U}_i = \mathcal{U}_i^G \times \mathcal{U}_i^D \times \mathcal{U}_i^P. \]

Taking into account the system elements dynamics and for a given time horizon \( N \) we can formulate the following dynamic optimal power flow (DOPF) problem as follows:

\[ \min_{x_i \in \mathcal{X}_i, u_i \in \mathcal{U}_i} \sum_{t=0}^{N} \sum_{i \in \mathcal{D}} P_i(t), \quad (57) \]
\[ \text{s.t.} \quad P_i(t) = P_i^G(t) + P_i^S(t) - P_{i}^D(t) \]
\[ = \sum_{j \in \mathcal{N}_i(t)} \left( \sqrt{2} g_{ij} v_j(t) - g_{ij} R_{ij}(t) \right) - b_{ij} T_{ij}(t), \quad \forall i \in \mathcal{V}, \]
\[ Q_i(t) = Q_i^G(t) + Q_i^S(t) - Q_{i}^D(t) \]
\[ = \sum_{j \in \mathcal{N}_i(t)} \left( -\sqrt{2} h_{ij} v_j(t) + b_{ij} R_{ij}(t) \right) - g_{ij} T_{ij}(t), \quad \forall i \in \mathcal{V}, \]
\[ 2v_i(t) v_j(t) \geq R_{ij}(t)^2 + T_{ij}(t)^2, \quad \forall (i,j) \in \mathcal{E}, \]
\[ R_{ij}(t) \geq 0, \quad \forall (i,j) \in \mathcal{E}, \]
\[ P_i(t) \leq P_{i}(t) \leq P_{\mathcal{V}}, \quad \forall i \in \mathcal{V}, \]
\[ Q_i(t) \leq Q_{i}(t) \leq Q_{\mathcal{V}}, \quad \forall i \in \mathcal{V}, \]
\[ \frac{V^2}{\sqrt{2}} \leq v_i(t) \leq \frac{\bar{V}^2}{\sqrt{2}}, \quad \forall i \in \mathcal{V}, \]
\[ x_i(t+1) = A_i x_i(t) + B_i u_i(t), \quad \forall i \in \mathcal{V}, \]
\[ x_i(t) \in \mathcal{X}_i, \quad u_i(t) \in \mathcal{U}_i, \quad \forall i \in \mathcal{V}. \]

This formulation of DOPF, as defined in (57), takes into account the dynamics of the distribution system. These dynamics allow a better insight of the system behavior. Simulations can be done in open or closed loop for a given time horizon. Closed loop simulations are consisted of receding horizon control, which basically means that at every time instant the DOPF is solved for a prediction horizon \( N \) but only first step of the control strategy is implemented. After the implementation of first step, the dynamic states are then updated and the calculations are repeated starting from the new current state.

VI. FUTURE RESEARCH

The search for the optimal reconfiguration of the power distribution system has been an important research topic in recent years. The report gives an overview of the work in the field of distribution system reconfiguration. As the topic is rather new, there are several areas for future research that could lead to better modeling and analysis of system behavior.

Firstly the more appropriate (e.g. compact) mathematical formulation of radiality constraint for meshed networks can be investigated. Radiality problem for planar networks is rather similar to the minimum spanning tree problem in graph theory. The problem is that the distribution grid today is built as
meshed network with more and more distributed generators installed which even more complicates the formulation of radiality constraints for DSR problem.

Further improvements in the model of the distribution grid can be done by including dynamical models of even more elements of the system. Dynamic model of transformers OLTC is addressed here as they are mostly used by the distribution system operators. Transformers with OLTC have the possibility to adjust and control the voltage level without stopping the power supply to the customers.

To efficiently solve to convex problems in OPF formulation the approximation of the SOCP offers a lot of research possibilities. Extension of OPF with binary values in order to achieve reconfiguration possibility results in MILP formulation. Further analysis can address the solution time of the DSR problem that is formulated as an MILP.

Parametric solution of DSR optimization problem may address economic objectives and be used for system planning. System planning can be done in everyday operation of the system or in order to preform large system analysis. Inclusion of different parametric objectives to DSR problem can gain even more benefit for the system operators.

Creation of testing scenarios for DSR and defining the baseline model is also very interesting topic for future work that needs detail research. The baseline model can insure the evaluation of the solution of DSR problem that will give the operator the possibility to choose the right actions while operating the system.

REFERENCES


